MATH 430, SPRING 2022
HOMEWORK 7, DUE FRIDAY APRIL 29

Problem 1. *Prove the Chinese Remainder Theorem: If* $d_1, ..., d_n$ *are relatively prime natural numbers, and* $a_1, ..., a_n$ *are such that for all* i, $a_i < d_i$, *then there is some* $c \in \mathbb{N}$, *such that for all* i, $c = a_i mod d_i$.

Problem 2. *Let Let* $f : \mathbb{N} \rightharpoonup \mathbb{N}$ *be the function* $f(x) = y$ *iff* x *codes a sequence of length* 2 *and* y *is the first element of that sequence. (Note that* f *is partial.) Show that* f *is recursive*

Problem 3. *For each of the following sequences, write down the number that codes them. You can leave your answer in a prime factorization form (i.e. there is no need to simplify)*

(1) $\langle 5, 0 \rangle$
(2) $\langle 0, 0, 1 \rangle$
(3) $\langle 1, 2, 1, 0 \rangle$

Problem 4. *For each of the following numbers, determine if they code a sequence. If yes, write down the sequence they code.*

(1) 140
(2) 90
(3) *1200*

For the next problem, recall that for $a \in \mathbb{N}$, ϕ_a is the formula coded by a. Suppose $\phi_{prov}(x, y)$ is a Σ_1 formula, such that

$$\mathfrak{A} \models \phi_{prov}[a, b] \text{ iff } PA \vdash \phi_a[b].$$

We will define this formula in class. Here $\phi_a[b]$ means that a codes a formula with one free variable, $\phi_a(x)$, and we are plugging in b for x.

Let e be the Gödel number of $\neg\phi_{prov}(x, x)$. In other words, $\phi_e = \neg\phi_{prov}(x, x)$. Define

$$\sigma := \neg\phi_{prov}(e, e).$$

Note that σ is exactly $\phi_e(e)$, and informally it says "I am not provable".

Problem 5. (1) *What is the complexity of* σ*?*
(2) *Show that* $\mathfrak{A} \models \sigma$ *iff* $PA \nvdash \sigma$.